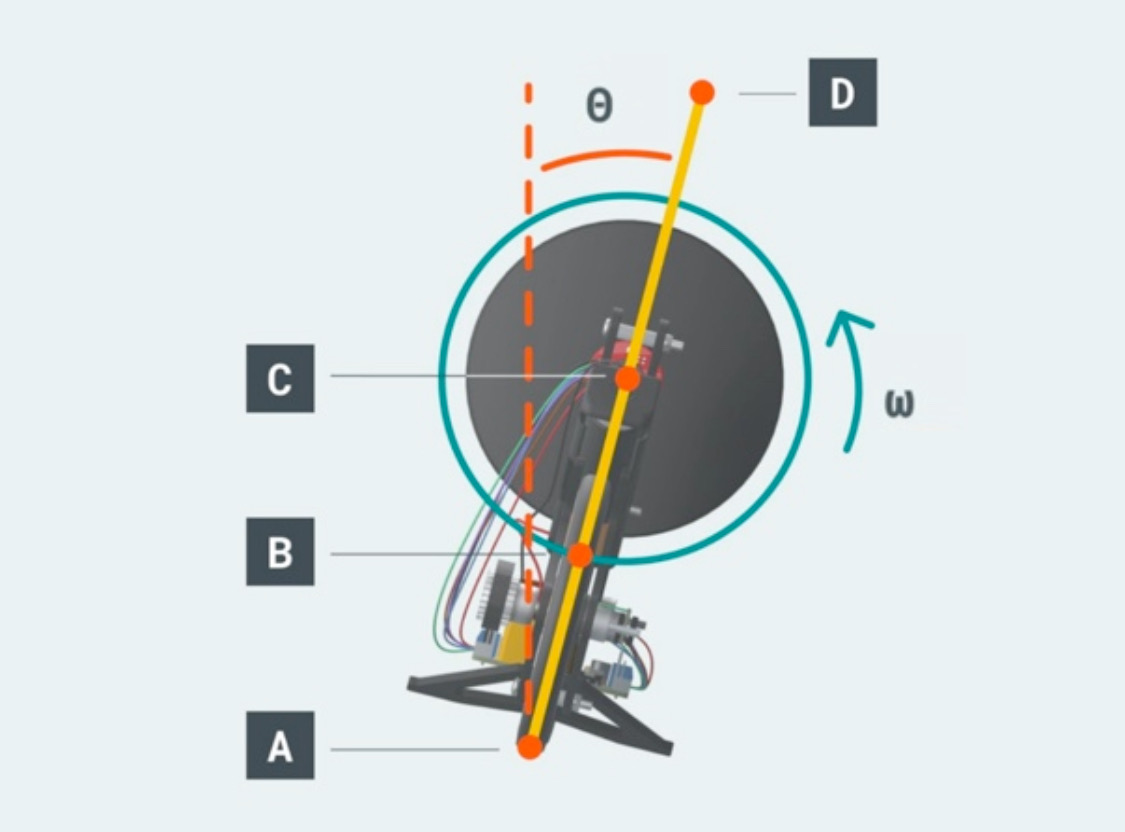
Principles of the motorcycle (Control)

Physical Model:



平衡摩托⻋的问题可以被认为是控制倒摆的问题。

The problem of balancing motorcycles can be considered a problem of controlling the inverted pendulum.

我们将摩托⻋建模为由两个关键部件组成的倒⽴摆：1. 通过具有⼀定⾃由度的旋转接头与地⾯相连的钟摆杆。2. 通过另⼀个具有⼀定⾃由度的旋转接头连接到钟摆杆的惯性轮。

We model the motorcycle as an inverted pendulum composed of two key components:

1. A pendulum rod connected to the ground through a rotary joint with a certain degree of freedom.

2. The inertia wheel connected to the pendulum rod through another rotary joint with a certain degree of freedom.

图中A是摆杆与地⾯之间旋转接头的旋转轴。是摆杆的⻓度，即摩托⻋框架的⻓度。B是摆杆的质⼼。C是摆杆与惯性轮之间转轴的旋转轴。

In the figure, A is the rotation axis of the rotary joint between the pendulum rod and the ground. Is the length of the pendulum rod, and is the length of the motorcycle frame. B is the quality of the pendulum. C is the rotation axis of the rotation axis between the pendulum rod and the inertia wheel.

我们设置变量：1. 摆杆倾斜⻆度，角速度。当钟摆完全直⽴时等于0度，当钟摆顺时针倾斜时等于正，当钟摆逆时针倾斜时等于负。

1. 惯性轮的转速和加速度。

We set the variables:

1. The rotational displacement of the inertia wheel , angular velocity , and angular acceleration . When the pendulum is completely straight, it is equal to 0 degrees, when the pendulum is tilted clockwise, it is equal to positive, and when the pendulum is tilted counterclockwise, it is equal to negative.

2. Rotational speed and acceleration of the inertia wheel.

**Model establishment:**

1) First, we analyze torques which are acting on the system.

Torque due to gravity force applied to the pendulum rod:

Torque due to gravity force applied to the inertia wheel:

Torques created by the inertia wheel motor:

2) Then we write down equations for moments of inertia of the key components.

Moment of inertia of the inertia wheel about point C:

By applying the parallel axis theorem, we get moment of inertia of the inertia wheel about point A:

Moment of inertia of the pendulum rod around its center

of mass B:

By applying the parallel axis theorem, we get moment of inertia of the pendulum rod about point A:

1. Next, we write the equations of motion.

Since torque and moment of inertia value are additive:

Which is:

Rotation of the inertia wheel is described by the following equation of motion:

1. Our next step is bringing the above equations of motion to state-space format. We will firstly define a state vector:

Finally, we derive the state space expression:

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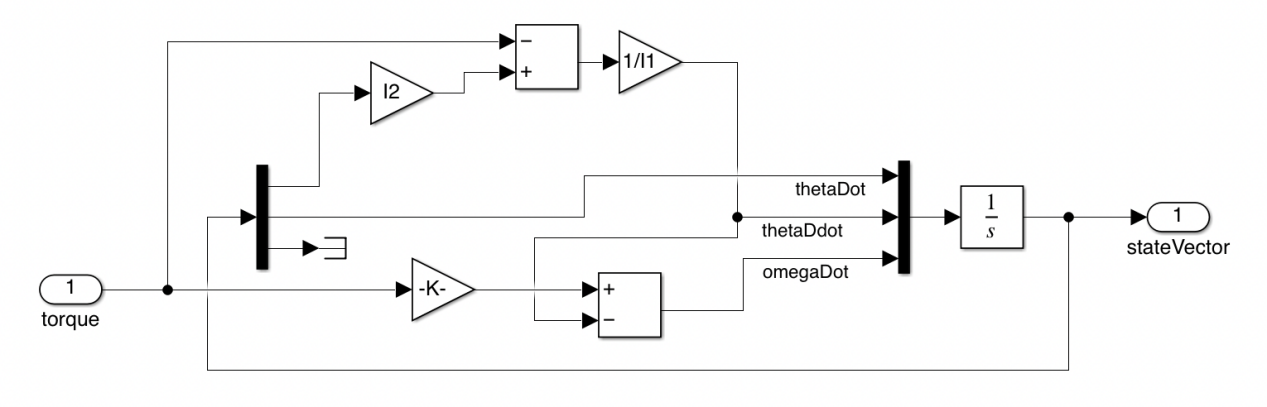
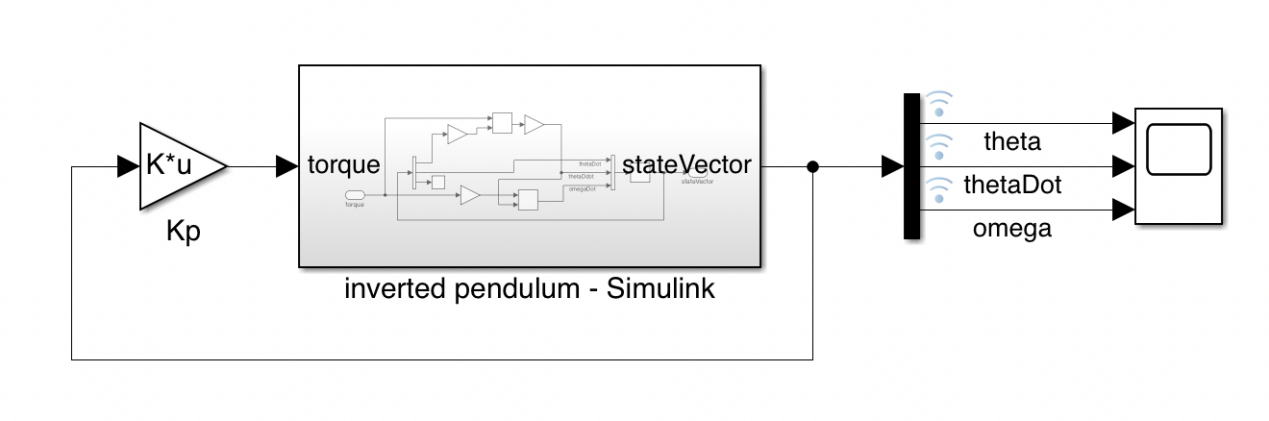
Therefore,

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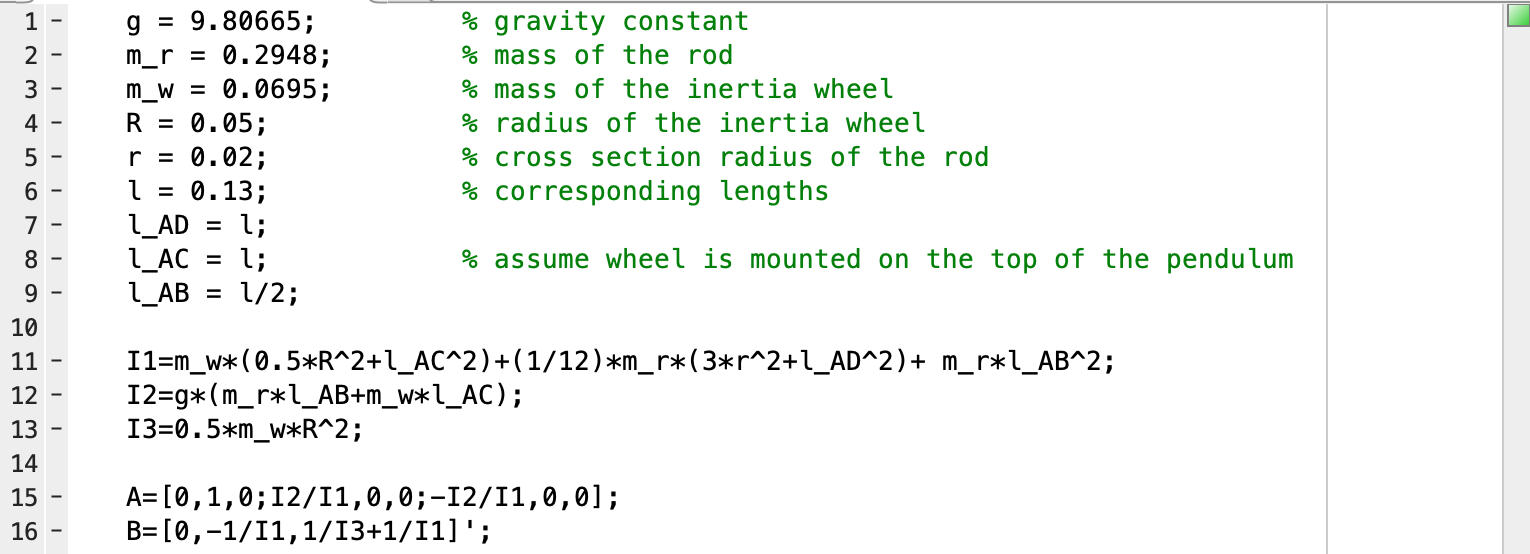
Our next step is implementing the above equation in software, to allow simulating the

behavior of this dynamical system and designing a feedback controller.

Controller in Simulink:



Model parameters:



The result:

